Closing Thu: Sup 8-9
Closing Fri/Sun: 1.1
Closing Tue: 1.2/3, 1.6(pt 1)

Entry Task:
Get out the paperweights graph.
Assume the market price ~~for~~ is 2.50 dollars/paperweight.

A) Draw TR.

B) What quantity maximizes profit?
What is the maximum profit?

C) What would the **MR graph** look like? Draw it with the MC, AC, AVC graphs.

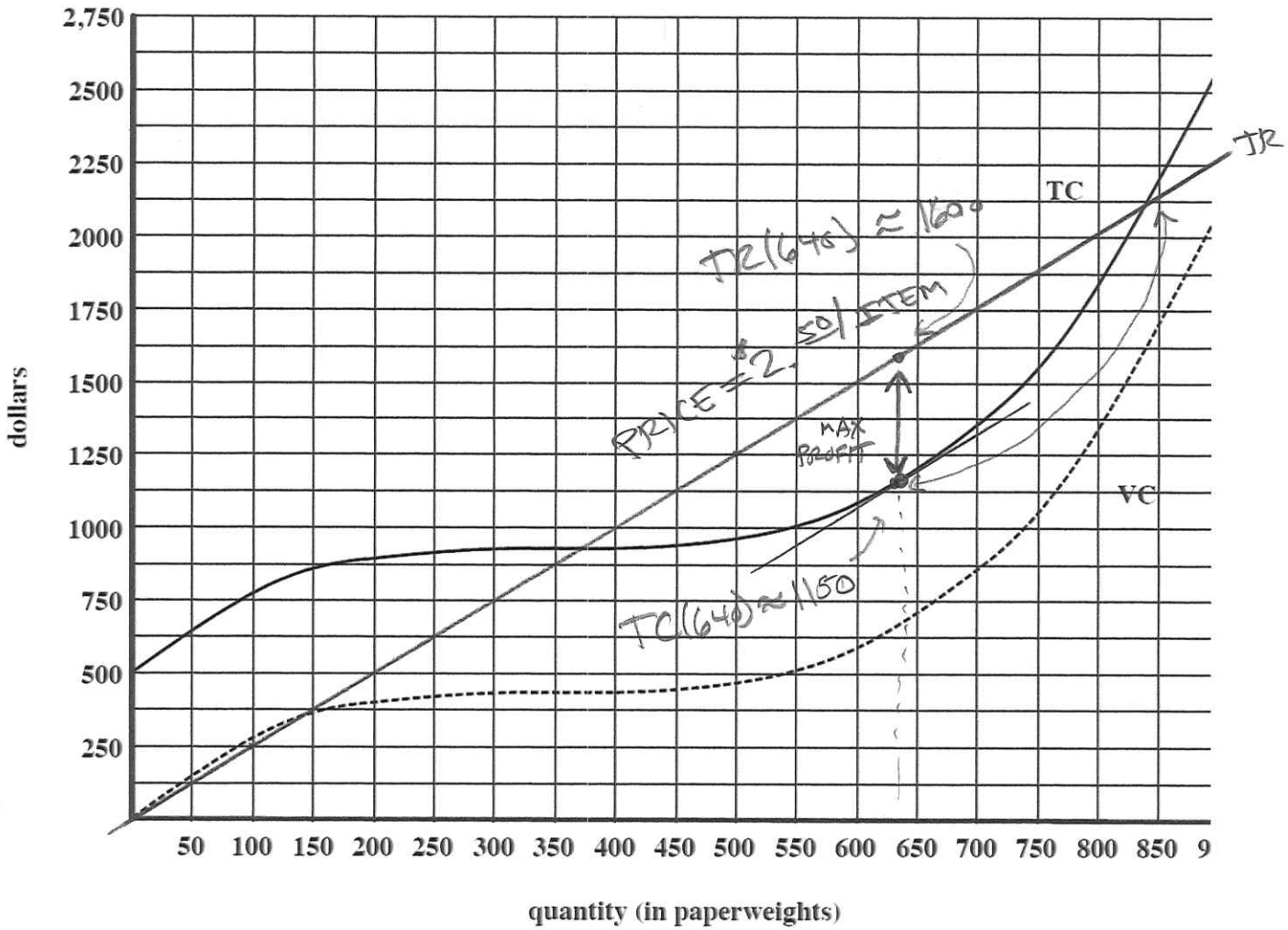
A PRICE = $2.50/ITEM.
⇒ TR IS A DIAGONAL LINE WITH SLOPE: 2.5
(0,0), (100,250), ..., (500,1250)...
(SEE NEXT PAGE)

B "LARGEST VERTICAL GAP WHEN TR IS ABOVE TC"
"MATCH SLOPES"
SLOPES MATCH AT ABOUT $q=640$

MAX PROFIT $= TR(640) - TC(640)$
$\approx 1600 - 1150 \approx$ 450 dollars

C MR = $2.50/ITEM ← ADDITIONAL REVENUE MONEY WHEN YOU SELL EACH ITEM
HORIZONTAL LINE!

(SEE GRAPH, FLIP TWO PAGES)
MAX PROFIT OCCURS WHEN MR=MC
(AND WHEN YOU SWITCH ~~FROM~~ MR>MC TO MR<MC)


2,750
2500
2250
2000
1750
1500
1250
1000
750
500
250
dollars
50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800 850 9
quantity (in paperweights)
TC
TR
VC
TR(640) ≈ 1600
$2.50/ITEM
PRICE = $2.
MAX PROFIT
TC(640) ≈ 1150

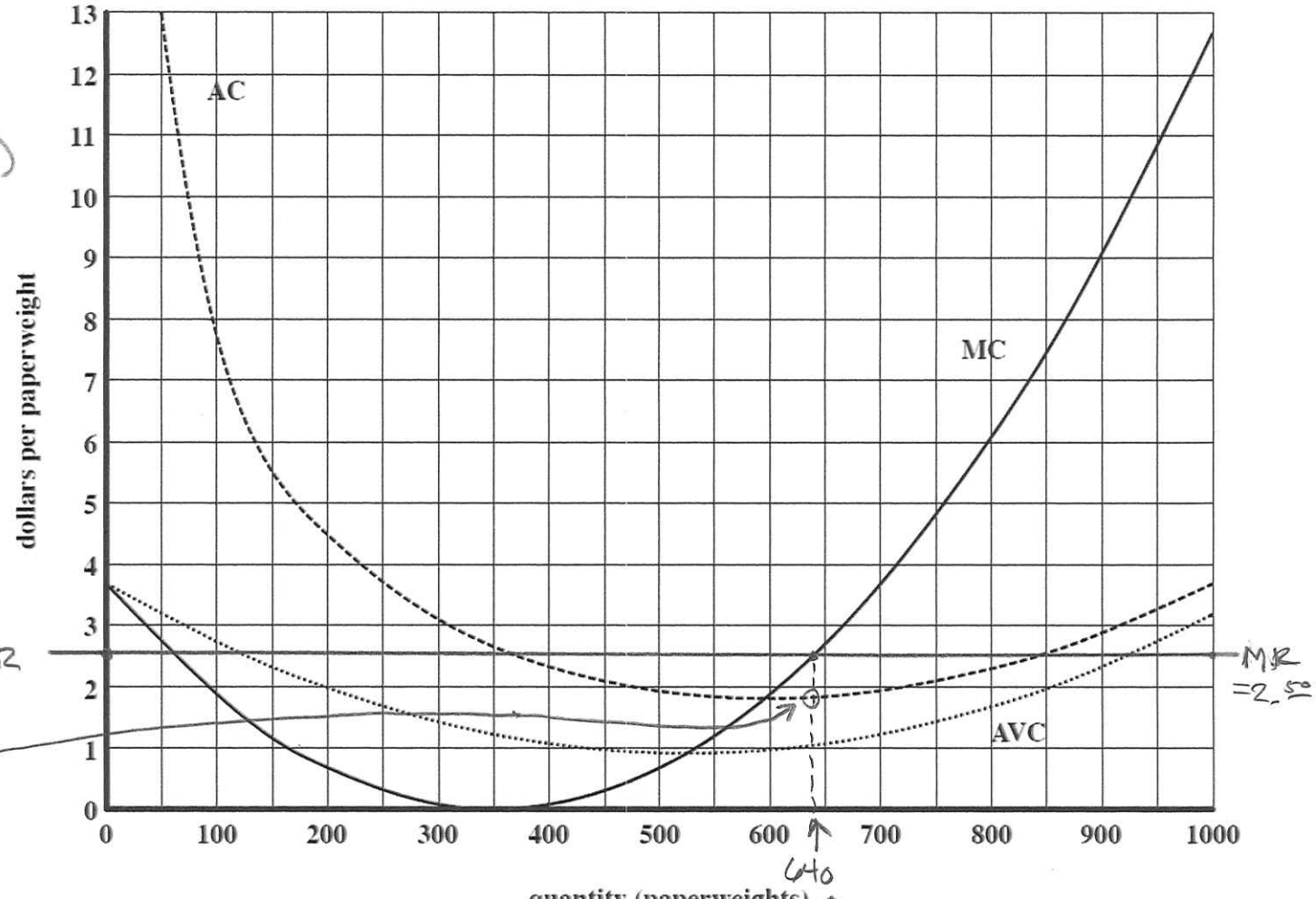
THIS IS HOW YOU DO THE LAST PART OF THE LAST PROBLEM ON SUPP. 8-9 HW!

SIDE NOTE

IT IS HARDER TO ESTIMATE TR(640) − TC(640) FROM THIS GRAPH, BUT STILL DOABLE BY USING OUR FORMULAS

① SINCE PRICE = \$2.50 AND QUANTITY = 640

$$TR(640) = 2.50 \times 640 \approx \$1600$$



② SINCE $AC(640) \approx 1.80$ AND $\underbrace{AC(640)}_{1.8} = \frac{TC(640)}{640}$

$$\Rightarrow 1.8 = \frac{TC(640)}{640}$$

$$\Rightarrow TC(640) \approx 1.8 \times 640 = 1152$$

MAX PROFIT OCCURS WHEN MR = MC HERE

So MAX PROFIT $= TR(640) - TC(640) \approx 1600 - 1152 = \boxed{\$448}$ ← CLOSE TO WHAT WE GOT WITH OTHER GRAPH!

Before we move on, let's have a
10-min Q & A on the business terms.
Do you understand how to...

On the TC, VC graphs:

1. Read/Use FC
2. Compute MC, AC, AVC
3. Given MC, AC or AVC, find q
4. Find BEP and SDP
5. Given a price, draw TR
6. Max profit

On the MC, AC, AVC graphs:

1. Compute TC or VC
2. Find BEP and SDP
3. Given a price, draw MR
4. Max profit

Section 1.1: Linear Equations (Skills)

Motivation: So far this entire course has been about **rates**. When we use a rate to predict the future with algebra, the function we get is a line.

TODAY: SOLVING SKILLS

NEXT LECTURE: LINEAR FUNCTIONS & APPLIED PROBLEMS

IN TWO LECTURES: TR, TC, VC, MC, MR, AC, AVC, ETC. WITH LINEAR FUNCTIONS

A collection of motivating examples...

Ex: Thru the first several games, Husky football had 124 points and averaged 49.5 points/game. If the rate continued, how many total pts would they have *x* games later?

Total points = 124 + 49.5x

Ex: For electricity I pay a flat fee of $30 per month plus $0.05 per kilowatt hour of use.

"payment for *x* kilowatt hours of use"

Payment = 30 + 0.05x

Ex: Your investment earns 9% interest each year.

Starting value = *x*

"interest from *x* (1 year)" = 0.09x

"total value (1 year)"= x + 0.09x

Value in 1 year = 1.09 x

Ex: The ST stock starts at $10 and increasing at a rate of $5/month

ST(x) = 10 + 5x = `value in *x* months'

Ex: Hats sell for $5 per item.

TR(x) = 5x.

We manufacture hats. Fixed costs equal $200 each day and it costs $3 to produce each hat.

TC(x) = 200 + 3x.

Terminology from Algebra

Def'n: A ***mathematical expressions*** is a formula involving letters, numbers & operations; *no equal sign*
Examples:

$$\frac{5x}{2} - 3,\ 10 + \sqrt{3+x},\ 5x - \frac{4}{x^{3/2}}$$

You don't solve expressions.

Def'n: An ***equation*** has an equal sign and a mathematical expression on <u>both</u> sides.
Examples of <u>linear equations</u>:

$3x + 4 = 10,\quad \frac{5x}{2} - 2 = 4x,$

$4\,(3 - 2x) = 16 + 5x$

You do solve equations!

Recall: *Functional notation* gives a name to a formula/expression. In that setting the equal sign is defining a rule/function. We are NOT solving

Examples of <u>linear functions</u>:

$ST(x) = 10 + 5x$

$TR(x) = 5x$

$TC(x) = 200 + 3x$

Note: These are just giving a functior a name to an expression.
You don't solve functions.

Solving Skills Practice:

LHS RHS

1. Solve $\frac{3x-1}{2} = 10.$ ×2

$2 \cdot \frac{(3x-1)}{2} = 2 \cdot 10$

$3x - 1 = 20$

$+1 \quad +1$

$\frac{3x}{3} = \frac{21}{3}$

$x = 7$

SIDE NOTE: $\frac{3x-1}{2} = \frac{1}{2}(3x-1)$ ⌐ SAME ¬

STEP 1: CLEAR DENOMINATORS (MULTIPLY BY 2)

STEP 2: ALL CONSTANTS TO SAME SIDE.

STEP 3: GET VARIABLE BY ITSELF.

CHECK!!!

$LHS = \frac{3x-1}{2} = \frac{3 \cdot 7 - 1}{2} = \frac{21-1}{2} = 10$

$RHS = 10$ ← SAME ✓

YES, WE KNOW WE ARE CORRECT!

You Try

2. Solve $\frac{3x}{4} - 2 = 3 - \frac{2}{3}\left(x - \frac{1}{2}\right)$

$\cdot 3$

$3 \cdot \frac{3}{4}x - 3 \cdot 2 = 3 \cdot 3 - 3 \cdot \frac{2}{3}\left(x - \frac{1}{2}\right)$

$\frac{9}{4}x - 6 = 9 - 2\left(x - \frac{1}{2}\right)$ ← DISTRIBUTED

$\frac{9}{4}x - 6 = 9 - 2x + 1$

$\frac{9}{4}x - 6 = 10 - 2x$

$\frac{9}{4}x = 16 - 2x$ $\cdot 4$

$4 \cdot \frac{9}{4}x = 4 \cdot 16 - 4 \cdot 2x$

$9x = 64 - 8x$

$17x = 64$

$\boxed{x = \frac{64}{17}}$ ← LEAVE IN THIS EXACT FORM IN WEBASSIGN (DO NOT GIVE DECIMAL

CHECK!!!!

$LHS = \frac{3}{4}\left(\frac{64}{17}\right) - 2 \approx 0.8235\ldots$

$RHS = 3 - \frac{2}{3}\left(\frac{64}{17} - \frac{1}{2}\right) \approx 0.8235\ldots$

SAME!!

3. Solve $\frac{10}{x-1} = 5$

$\cdot (x-1)$

$(x-1) \cdot \frac{10}{(x-1)} = (x-1) \cdot 5$

$10 = 5x - 5$ $\quad +5$

$15 = 5x$ $\quad \div 5$

$\frac{15}{5} = x$

$\boxed{x = 3}$

CHECK!!!

$LHS = \frac{10}{3-1} = \frac{10}{2} = 5$

$RHS = 5$ ← SAME ✓

✓✓

4. Solve for t in the equation

$S = P + At$

$S - P = At$

$\boxed{t = \frac{S-P}{A}}$

CHECK!!

$LHS = S$

$RHS = P + A\left(\frac{S-P}{A}\right)$ ← SAME ✓

$= P + S - P = S$ ↵

Inequalities

We simplify inequalities the **same** way we solve equations, except *if you multiply or divide by a negative then you flip the inequality.*

Ex $1 < 3$ ← IF YOU MULTIPLY BOTH SIDES BY -4, THEN YOU GET $-4 > -12$ ← HAS TO FLIP!

Examples:

1. Simplify $-2x + 4 > 10$

-4

$\Rightarrow \quad -2x > 6$

$\div(-2)$ ← FLIP!

$x < \frac{6}{-2}$

$\boxed{x < -3}$

CHECK:

RANDOMLY PICK SOME $x < -3$

DOES IT WORK IN THE ORIGINAL INEQUALITY?

$x = -4 \Rightarrow$ LHS $= -2(-4) + 4 = 12$

RHS $= 10$

$12 > 10$ YES ✓

2. Simplify $0.5(x - 4) < 0.1x + 3$

$0.5x - 2 < 0.1x + 3$ $+2$

$0.5x < 0.1x + 5$ $-0.1x$

$0.5x - 0.1x < 5$

$0.4x < 5$ $\div 0.4$

$x < \frac{5}{0.4}$

$\boxed{x < 12.5}$

CHECK:

TRY $x = 12$

LHS $= 0.5(12 - 4) = 4$

RHS $= 0.1x + 3 = 4.2$

$4 < 4.2$ ✓

Some Application Problems:

Example: Cupcake Business

FC = \$300,

selling price = \$1.50/cupcake

production costs = \$0.50/cupcake

- At what quantity will the profit be zero? (i.e. break even quantity).
- At what quantity will profit be \$450?

$TR(q) = \text{PRICE} \cdot \text{QUANTITY} = 1.50 \cdot q$

$\boxed{TR(q) = 1.5q}$

$VC(q) = (\text{PRODUCTION COST PER ITEM}) \cdot \text{QUANTITY} = 0.50 \cdot q$

$\boxed{TC(q) = FC + VC(q) = 300 + 0.5q}$

PROFIT $= 0 \Rightarrow TR - TC = 0$

$\Rightarrow TR = TC$

WANT $1.5q \overset{?}{=} 300 + 0.5q$

$-0.5q \qquad -0.5q$

$1q = 300$

$\boxed{q = 300}$

CHECK! $TR(300) = \$450$

$TC(300) = \$450$

PROFIT $= 450 - 450 = \$0$ ✓

WANT PROFIT $\overset{?}{=} 450$

$\Rightarrow TR - TC = 450$

$\Rightarrow (1.5q) - (300 + 0.5q) \overset{?}{=} 450$

↑ VERY IMPORTANT

$\Rightarrow 1.5q - 300 - 0.5q = 450$

$\Rightarrow 1.5q - 0.5q = 750$

$\Rightarrow 1q = 750$

$\Rightarrow \boxed{q = 750}$

CHECK! $TR(750) = \$1125$

$TC(750) = \$675$

PROFIT$(750) = \$450$ ✓✓

Directly from homework:

10) A retired woman has $240,000 to invest. She has chosen one relatively safe investment fund that has an annual yield of 9% and another, riskier fund that has a 13% annual yield. How much should she invest in each fund if she would like to earn exactly $24,000 per year from her investments?

LET x = "AMOUNT INVESTED IN 'SAFE' FUND"

AND y = "AMOUNT INVESTED IN 'RISKY' FUND"

FACT 1: $x + y = \$240{,}000 \Rightarrow y = 240000 - x$

FACT 2: $0.09x + 0.13y = \$24{,}000$

COMBINING FACT 1 AND FACT 2 GIVES

$$0.09x + 0.13(240{,}000 - x) = 24{,}000$$

$$0.09x + 31200 - 0.13x = 24000$$

$$31200 - 0.04x = 24000$$

$$31200 = 24000 + 0.04x$$

$$7200 = 0.04x$$

$$\frac{7200}{0.04} = x$$

$$\boxed{x = 180{,}000}$$

$$\boxed{y = 240000 - x = 60{,}000}$$

INVEST \$180,000 IN SAFER ACCOUNT AND \$60,000 IN RISKY ACCOUNT!

NOTE: $0.09x$ = "INTEREST FROM 'SAFE' FUND"

$0.13y$ = "INTEREST FROM 'RISKY' FUND"

TOTAL INTEREST IN ONE YEAR $= 0.09x + 0.13y$

CHECK!

$$180000 \cdot 0.09 + 60000 \cdot 0.13$$

$$= 24000 \checkmark$$